Allen and Gale on risk-taking and competition in banking

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Abstract

Using an elegant simple model, Allen and Gale [Comparing Financial Systems. MIT Press. (2001)] obtain a result with significant policy implications: portfolio risk of banks increases as competition in banking, measured by the number of banks, increases. That result is, however, lacking in robustness. If banks play a game not all that different from that assumed by Allen and Gale, then, we show, a markedly different result obtains: risk-taking by banks is independent of the number of banks.

Keywords: concentration and risk-taking in banking.

1 Introduction

In Allen and Gale (2001, Chapter 8), the authors (hereafter AG) provide an answer to a policy question of considerable interest: How are the structure and the fragility of a banking industry related? Their answer, supported by a variety of arguments, is that equilibrium risk-taking by banks, hence industry fragility, is increasing in the number of banks, their proxy for the intensity of competition. In this brief note, we focus on the simplest of those arguments (pp. 242-50), in which use is made of a game played by a deposit insurer and n banks, alike in all relevant respects. We are not convinced that partial equilibrium analysis can produce a satisfactory answer to the question that AG address. But here our essential point is that their result is lacking in robustness. For a different game – arguably as reasonable as theirs – equilibrium bank risk-taking is independent of the number of banks. We first describe the AG game and give their result and then go on to describe our game, a modification of theirs, and establish the existence of a unique equilibrium characterized by the noted independence.

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2 The Allen-Gale game

AG introduce their deposit insurer with the following passage (p. 248):

> We assume that the premium for deposit insurance is set before the banks choose their strategies and that it is the same for each bank, independent of the strategy chosen. In equilibrium, the premium accurately reflects the cost of deposit insurance provided by a risk-neutral insurer.

By our reading, the insurer is a risk-neutral behavioral player or institution that, like a Stackelberg leader, plays first.[1] As AG implicitly assume, it knows the objective functions and therefore equilibrium strategies of all n banks. It is therefore able, at the beginning of the game, to announce a per dollar deposit premium π such that at equilibrium it breaks even.

In the analysis of AG with which we are concerned, they are interested only in symmetric equilibria. Accordingly, we assume that for any n the n banks are alike in all ways that count. Let d be a representative bank's deposit total and, since it has no equity, its supply of funds for investment. Suppose, as AG do, that its portfolio is without idiosyncratic risk and further that it earns gross portfolio rates of return y and 0 with probabilities $p(y)$ and $1 - p(y)$. Its objective or expected profit function is then

$$p(y)[y - R(D/n) - \pi]d \tag{1}$$

where $D = \Sigma_{i=1}^{n} d = dn$, the function $p : [0, \bar{y}] \to [0, 1]$ is C^2 with $p' < 0$, $p'' \leq 0$ and such that $p(0) = 1$ and $p(\bar{y}) = 0$ and the function $R : \mathbb{R}_+ \to \mathbb{R}_+$ is C^2 with $R', R'' > 0$ and such that $R(0) = 0$ and $R(\infty) = \infty$.[2] The representative bank maximizes (1) by choice of a strategy $(y, d) \gg 0$, subject to the given (previously announced) premium π. The first-order conditions are

$$p'(y)[(y - R(D/n) - \pi] + p(y) = 0$$

and

$$y - R(D/n) - \pi - \frac{R'(D/n)d}{n} = 0.$$

If any exist, the solutions $(\hat{y}, \hat{d})$ for the n banks constitute positive and symmetric Nash equilibria (hereafter simply equilibria).

[1] The n banks are Cournot competitors. From p. 243: "We use a model of Cournot competition, in which banks choose the volume of deposits they want, subject to an upward sloping supply of funds [deposits] schedule."

[2] AG (p. 246) get to a perfectly competitive deposit or supply-of-funds market in the "normal" way, by letting n and the number of consumers (depositors) m increase in proportion and without bound. Thus, if the representative bank goes from being one of n to being one of $2n$ banks, its deposit supply function does not change. So it is legitimate as well as convenient to assume, as AG do, that the gross deposit rate of return R depends on deposits per bank D/n.

For AG (p. 248), π is such that at equilibrium "the expected return on deposits [equals] the return demanded by depositors." That is, π satisfies

$$p(y)[\pi + R(D/n)] = R(D/n)$$

for equilibrium values of y and D, or equivalently

$$\pi = \frac{1 - p(y)}{p(y)} R(D/n). \tag{2}$$

That equality can also be derived by making use of the assumption (p. 248) that π "accurately reflects the cost of deposit insurance provided by a risk-neutral insurer" or, as we interpret that passage, π must be such that the insurer's expectation is that at equilibrium it breaks even. Its expected revenue is $[p(y)d]\pi$ and its expected payout is $[1 - p(y)]R(d)d$.

With (2), the first-order conditions are

$$p'(y)[y - \frac{R(d)}{p(y)}] + p(y) = 0 \tag{3}$$

and

$$y - \frac{R(d)}{p(y)} - R'(d)d/n = 0. \tag{4}$$

Now, let $(y^n, d^n) \gg 0$ be an equilibrium for n banks and suppose with AG that $(y^n, d^n) \to (y^0, d^0)$ as $n \to \infty$. If d^0 is finite, then $R'(d^n)d^n/n \to 0$ as $n \to \infty$; and, as follows from (4),

$$\lim_{n \to \infty} y^n - \frac{R(d^n)}{p(y^n)} = 0.$$

Thus, from (2), $p(y^n) \to 0$ as $n \to \infty$ and therefore $y^0 = \bar{y}$.

That completes AG's proof of their conclusion that banking industry fragility is increasing in the number of banks – or, assuming that an increase in the number of banks means increased competition, that the fragility of the banking industry increases as competition among banks becomes more intense. But note that there is nothing in AG's argument that goes to the existence of an equilibrium of their game.

3 An alternative game

It is in what we assume about the deposit insurer that we part company with AG. For us, the deposit insurer announces not a deposit insurance premium, but rather at the start of play a rule indicating what premium the bank that chooses the pair (y, d) will pay:

$$\pi = \frac{1 - p(y)}{p(y)} R(D/n), \tag{5}$$

where y and d are costlessly observed by the insurer. So the bank has to take into account that its premium will change as its strategy changes. It maximizes

$$p(y)[y - R(D/n) - \pi]d$$

by choice of a strategy $(y, d) \gg 0$, but subject to (5) rather than a given premium π. Equivalently, it maximizes

$$[p(y)y - R(D/n)]d$$

by its choice of strategy $(y, d) \gg 0$.

Each bank's equilibrium strategy satisfies

$$p'(y)y + p(y) = 0 \tag{6}$$

and

$$p(y)y - R(d) - \frac{R'(d)d}{n} = 0. \tag{7}$$

From (6), the optimal y satisfies

$$y = -\frac{p(y)}{p'(y)}. \tag{8}$$

So if there is an equilibrium portfolio risk $\hat{y}$, it is independent of the number of banks.

To complete our argument, we show the existence of a unique equilibrium: that is, for each $n \geq 1$, a unique pair $(\hat{y}, \hat{d}_n) \in [0, \bar{y}] \times \mathbb{R}_{++}$ that solves (6) and (7). Let $r(y) = -p(y)/p'(y)$. By AG's characterization of the function p: (i) $r(y)$ is continuous and strictly decreasing; (ii) $r(y)$ converges to a nonnegative limit as $y \to 0$; and (iii) $r(\bar{y}) = 0$. So there exists a unique value of $y \in (0, \bar{y})$, denoted $\hat{y}$, satisfying (8).

From condition (7) at $y = \hat{y}$,

$$R(d) + R'(d)\frac{d}{n} = p(\hat{y})\hat{y}, \tag{9}$$

where $p(\hat{y})\hat{y} \in (0, \bar{y})$. Let

$$\mathcal{L}(d, n) = R(d) + R'(d)\frac{d}{n}.$$

It follows from AG's characterization of the function R that, for each $n \geq 1$, $\mathcal{L}(d, n)$ is continuous, strictly increasing and such that $\mathcal{L}(0, n) = 0$ and $\lim_{d\to\infty} \mathcal{L}(d, n) = \infty$. So, for each $n \geq 1$, there exists a unique value of $d \in \mathbb{R}_{++}$, denoted $\hat{d}_n$, satisfying (9). Thus, with AG's assumptions about the functions p and R, there exists a unique (symmetric and strictly positive Nash) equilibrium for every $n \geq 1$.

References

Allen, Franklin and Douglas Gale, 2001. Comparing Financial Systems. Cambridge, MA.: MIT Press.